Art. XIV.—*On the Súrya Siddhánta, and the Hindú Method of Calculating Eclipses.* By William Spottiswoode, M.A., F.R.S., &c.

[*Read 19th January*, 1863.]

Some years ago it was suggested to me to undertake an edition and translation of the Súrya Siddhánta; but a long and careful study of the work convinced me that it would be impossible satisfactorily to accomplish the object without more assistance than was to be found in Europe. The MSS. were few and insufficient in accuracy; the lexicon was deficient in most of the technical terms; and the only works that threw any light upon the subject were those of Colebrooke, invaluable as far as they went, and the Abbé Guérin's Astronomie Indienne. The other writers who had touched upon the subject merely reproduced what was to be found in Colebrooke.

Mr. Hall's edition in the Bibliotheca Indica, and Mr. Burgess' elaborate translation and notes, published by the American Oriental Society,[1] now seem to leave little wanting upon the subject. But for those who wish to comprehend the nature, and estimate the real value, of the Hindú astronomical methods, without entering more deeply than necessary into the complexities of either text or commentary, it may still be useful to present the processes in as compendious a form as possible. I have therefore attempted to translate into modern mathematical language and formulæ the rules of the work in question.[2] The text, like all Sanskrit works, gives no account whatever of its rules or methods; and as the explanations of the commentators, being of comparatively recent date, have little or no interest for the history of the science, it appeared to me foreign to the present purpose to retain them. Under these circumstances I have contented myself in the case of exact formulæ, with occasionally adding a few of the steps necessary for verification; and in the case of approximate, with pointing out what assumptions are involved in the rules which they express. It should be added that, the assumptions so indicated are not

[1] Journal of the American Oriental Society, vol. vi, pp. 145—498.

[2] These rules are contained in Chaps. I—VI. To the remaining chapters the process is not applicable.

necessarily identical with, although in some sense equivalent to, those originally made by the author of the treatise.

From such observations as they were able to make, the Hindús deduced values for the mean motions of the sun, moon, and planets, supposed to revolve about the earth, and of their apsides and nodes. By means of these values they calculated back to remote epochs, when, according to their data, there would have been a general conjunction of parts or of the entire system.

The following is an outline of the process of calculating a lunar eclipse. First find the number of days elapsed from the commencement of the age, or period, to the mean midnight next before the full moon for which an eclipse was to be calculated. The original determination of the mean motions had of course given the current year of the period. This being done, an easy arithmetical process gave the mean longitude of the sun, the moon, and the moon's apsis.

The process of correction, whereby the true longitudes were thence deduced, is curious and peculiar. It had been noticed that the apsides, or points of slowest movement, and the positions of conjunction with the sun had proper motions. These were attributed to influences residing in the apsides and conjunctions respectively; and corrections due to each were accordingly devised. The undisturbed orbit was considered a circle with the earth (E) in the centre; and upon it the centre of a smaller circle or epicycle moved with a uniform angular velocity, equal, but opposite in direction, to that of the undisturbed planet; so that M being the centre, and m any given point on the epicycle, M m always remained parallel to itself. If then at the apsis, or conjunction (according as the correction of one or of the other was being calculated), M m was in a straight line with E m, the true position of the planet was conceived to be at the point where E m cut the undisturbed orbit. The radius moreover of the epicycle was variable; and its magnitudes at the odd and even quadrants being determined so as to satisfy observation, its intermediate variation was considered proportional to the sine of the mean anomaly.

The true longitudes and daily motions of the sun and moon having been found, the interval between mean midnight and the end of the half month, or moment of opposition in longitude, or middle of the eclipse is then determined. But since the Hindú time is reckoned from true sunrise to true sunrise, it is next required to determine the interval between mean midnight and true sunrise. This is effected by means of (1) the equation of time, found by a

simple but rather rough method; (2) the precession, of which more below; and (3) the ascensional difference.

The diameters of the sun, moon, and shadow, are found on the principle that their true are to their mean diameters, as their true are to their mean motions.

Lastly are determined the moon's latitude at the middle of the eclipse; the amount of greatest obscuration; the duration of the eclipse; of total obscuration (if it be total); and the times of first and last contact of immersion and emergence; by methods which do not require particular notice apart from the details themselves.

SURYA SIDDHANTA.

Chapter I.

On the Mean Motions of the Planets.

The divisions of time are as follow (vv. 11, 12):—

10 long syllables	=	1 respiration (= 4 seconds),
6 respirations	=	1 vinádí,
60 vinádí	=	1 nádí,
60 nádís	=	1 day.

Those of the circle are the same as ours (v. 28).

The civil day is reckoned from sunrise to sunrise, and for astrological purposes a month consists of 30 such days, and a year of 12 such months.

The astronomical day is the interval from midnight to midnight.

The lunar month is the interval from one new, or full, moon to the next. It is divided into 30 lunar days, which of course do not correspond with civil days. The lunar month is named after the solar month in which it commences. When two lunar months begin in the same solar month, the former is called an intercalary month, and both bear the same name. The civil day is named after the lunar day in which it ends; when two lunar days end in the same civil day, the former is "omitted," and both bear the same name (see also vv. 34–40).

The solar year is sidereal, and the zodiac is divided into 12 signs, to each of which corresponds a solar month (vv. 12, 13).

The following is the composition of the "Great Age," an imaginary period (vv. 15–17):—

	Solar Years.	Solar Years.
Dawn	144,000	
Kṛita Yuga	1,440,000	
Twilight	144,000	
Total ..		1,728,000
Dawn	108,000	
Tretá Yuga	1,080,000	
Twilight	108,000	
Total ..		1,296,000
Dawn	72,000	
Dvápara Yuga ..	720,000	
Twilight	72,000	
Total ..		864,000
Dawn	36,000	
Kali Yuga	360,000	
Twilight	36,000	
Total ..		432,000
Total of Great Age		4,320,000

Furthermore, the Kalpa (कल्पः) is thus composed (v. 18, 19):—

	Solar Years.	Solar Years.
Dawn		1,728,000
71 Great Ages ..	306,720,000	
1 Twilight	1,728,000	
1 Manvantara ..	308,448,000	
14 Manvantaras		4,318,272,000
1 Kalpa		4,320,000,000

The Kalpa is a day of Brahma. His night is of the same length; and his age consists of 100 years, each of 360 such days and nights. The total duration is 311,040,000,000,000 solar years (vv. 20, 21).

The following is a computation of the time from the commencement of the Kalpa to the end of the present Treta Yuga (vv. 21–23):—

	Solar Years.
Dawn of current Kalpa	1,728,000
6 Manvantaras	1,850,688,000
27 Great Ages	116,640,000
Treta Yuga	1,728,000
	1,970,784,000
But from the elapsed portion of the present Kalpa there must be deducted the time occupied in creation (v. 24, see also vv. 45–47)	17,064,000
	1,953,720,000

In their daily westward motion the planets lag behind the fixed stars each by the same absolute mean distance, viz., 11,858,717 yojanas (योजनाः); and their angular motion is inversely as the radius of the orbit. The initial point of the sphere is the end of the constellation Revatí (vv. 25–27).

[The principal star of Revatí is said to be 10′ W. of the above-mentioned point, and is supposed to be ζ Piscium.]

The numbers of revolutions of the planets, &c., are as follow (vv. 29–34, 41–44):—

	In a Great Age. Revolutions of the Planets.	In a Kalpa. Apsides.	Nodes.
Sun	4,320,000 ..	387	
Mercury	17,937,060 ..	368	488
Venus	7,022,376 ..	535	903
Mars	2,296,832 ..	204	214
Jupiter	364,220 ..	930	174
Saturn	146,568 ..	39	662
Moon:—			
Sidereal rev.	37,753,336		
Apsis	488,203		
Node	232,338		

From the foregoing data the following results are deduced (vv. 34–40):—

	In a Great Age.
Sidereal days	1,582,237,828
Deduct solar revolutions	4,320,000
Civil days	1,577,917,828

Sidereal solar years	4,320,000
	12
Solar months	51,840,000
Moon's sidereal revolutions	57,753,336
Deduct solar revolutions	4,320,000
Synodical revolutions (lunar months) ..	53,433,336
Deduct solar months	51,840,000
Intercalary months	1,593,336
Lunar months × 30 = lunar days ..	1,603,000,080
Deduct civil days	1,577,917,828
Omitted lunar days..	25,082,252

In order to find the number of civil days that have elapsed since the creation, or any other given epoch, to a given date, proceed first as in vv. 23, 24. Then (vv. 48–51) let—

Y = No. of years to end of last Kṛita Yuga,
y = " since Y,
m = complete solar months since y,
d = lunar days elapsed of current month.

Then the required number of lunar days

$$= 30\Big(12\,(Y + y) + m\Big)\Big(1 + \frac{1593336}{51840000}\Big) + d$$

$$= \Big(12\,(Y + y) + m\Big)\frac{2226389}{72000} + d.$$

And the corresponding number of civil days

$$= \Big\{\Big(12\,(Y + y) + m\Big)\frac{2226389}{72000} + d\Big\}\Big(1 - \frac{25082252}{1603000080}\Big)$$

$$= \Big\{\Big(12\,(Y + y) + m\Big)\frac{2226389}{72000} + d\Big\}\frac{394479457}{400750040}.$$

Suppose the planets were arranged in the order of their supposed distance from the Earth, viz., Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon. The first hour of the first day of the first month of the first year was assigned to the Sun; and so on for all the

other hours in the order given above. The succession for the days, months, and years will consequently fall as follows:—

Hours.	Days.	Months.	Years.
1	$1 = 7 \times 0 + 1$	$1 = 7 \times 0 + 1$	$1 = 7 \times 0 + 1$
2	$25 = 7 \times 3 + 4$	$31 = 7 \times 4 + 3$	$361 = 7 \times 51 + 4$
3	$49 = 7 \times 7$	$61 = 7 \times 8 + 5$	$721 = 7 \times 103$
4	$73 = 7 \times 10 + 3$	$91 = 7 \times 13$	$1081 = 7 \times 154 + 3$
5	$97 = 7 \times 13 + 6$	$121 = 7 \times 17 + 2$	$1441 = 7 \times 205 + 6$
6	$121 = 7 \times 17 + 2$	$151 = 7 \times 21 + 4$	$1801 = 7 \times 257 + 2$
7	$145 = 7 \times 20 + 5$	$181 = 7 \times 25 + 6$	$2161 = 7 \times 308 + 5$

Hence the following scheme of arrangement of planets, days, months, and years:—

	Hours.	Days.	Months.	Years.
1	Sun	Sun	Sun	Sun
2	Venus	Moon	Mars	Mercury
3	Mercury	Mars	Jupiter	Saturn
4	Moon	Mercury	Saturn	Mars
5	Saturn	Jupiter	Moon	Venus
6	Jupiter	Venus	Mercury	Moon
7	Mars	Saturn	Venus	Jupiter

Hence, to find the planet of the day we have only to find the remainder of D (or the number of days) $\div$ 7, and the planet opposite the corresponding place in the days column will be the planet required. Now the positions of the planets in the day column being of the form D, those in the month column are of the form $2\,D + 1$, and those in the year column of the form $3\,D + 1$. Hence, q implying quotient, and r remainder, the formulæ for finding the planet corresponding to a given—

$$\text{Day will be } \left(\frac{D}{7}\right)_r,$$

$$\text{Month ,, } \left(\frac{2\left(\frac{D}{30}\right)_q + 1}{7}\right)_r,$$

$$\text{Year ,, } \left(\frac{3\left(\frac{D}{360}\right)_q + 1}{7}\right)_r.$$

The mean place of the planets at any given time (No. of days elapsed = D) is given by the formula $\frac{DR}{C}$, where—

R = No. of revolutions in an age (vv. 53–4),
C = No. of civil days in an age.

To find the current year of the cycle of Jupiter (60 years); if J be the number of past revolutions and signs of Jupiter—

$$\text{Current year} = \left(\frac{J}{60}\right)_r.$$

The above method gives the mean places of the planets, &c., for the prime meridian (through Ujjayiní); we now proceed to find them for any other meridian (vv. 59, 60)—

Let ρ = radius of Earth = 1600 yojanas,
l = latitude.

Then $\rho\sqrt{10}$ = circumference of equator,
$r\sqrt{10}\cos. l$ = circumference of parallel, whose latitude is l.

Let t, t' be the calculated and observed lines of immersion and emersion of a total lunar eclipse, then the correction for longitude (and latitude) expressed in yojanas (vv. 63–65)

$$= \frac{(t \sim t')\, r\sqrt{10}\cos. l}{60} = (t \sim t')\cos. l\,\frac{80\sqrt{10}}{3}$$

And if n be the planet's mean daily motion, the mean position for the meridian of the place will be—

$$\frac{DR}{C} \pm \frac{n\,(t \sim t')\, r\sqrt{10}\cos. l}{60r\sqrt{10}\cos. l} = \frac{DR}{C} \pm n\,\frac{t \sim t'}{60}$$

And, if t'' = time before or after midnight expressed in nádís, then the planet's mean position at that time will be expressed by (v. 67)—

$$\frac{DR}{C} \pm \frac{nt''}{60}$$

The orbits are however inclined to the elliptic as follow (vv. 68–70):—

	°	′
Moon	4	30
Mars	1	30
Mercury	2	0
Jupiter	1	0
Venus	2	0
Saturn	2	0

CHAPTER II.

ON THE TRUE MOTIONS OF THE PLANETS.

The planets are advanced or retarded, or diverted in latitude, in various degrees, from their mean positions, by agencies situated in their apsides, nodes, &c. (vv. 1-14). To determine their true positions, a Table of Sines is necessary. The intervals of arc for which the sines are calculated are 225′; then, if s, s', s'', be the sines of 225′, 2×225′, 3×225′, we have the following rule for calculation :—

$$s = 225'$$

$$s' = s + s - \frac{s}{s}$$

$$s'' = s' + s - \frac{s}{s} - \frac{s'}{s}$$

And the Table given as the result of these formulæ (although from the 7th to the end some modifications have been made) is (vv. 15-27) :—

Arc.		Sine.	Arc.		Sine.	Arc.		Sine.
°	′	′	°	′	′	°	′	′
3	45	225	33	45	1910	63	45	3084
7	30	449	37	30	2093	67	30	3177
11	15	671	41	15	2267	71	15	3256
15	0	890	45	0	2431	75	0	3321
18	45	1105	48	45	2585	78	45	3372
22	30	1315	52	30	2728	82	30	3409
26	15	1520	56	15	2859	86	15	3431
30	0	1719	60	0	2978	90	0	3438

It must be remembered that the sine is a line, not a ratio, and consequently that sin. 90° ≐ radius.

The sine of any arc not an exact multiple of 225′ is given by the formula (vv. 31-33)—

$$\text{sin.}\ (n\ 225' + \theta) = \frac{\theta\ (\text{sin.}\ (n+1)\ 225' - \text{sin.}\ n\ 225')}{225'}.$$

Similarly the arc might be found from the sine.

If S be the Sun, N its node, and N S H a spherical triangle right angled at H, the Sun's declination D (= S H) is given by the equation (v. 28):—

$$\text{R sin. D} = \text{sin. S N sin. S N H}.$$

The corrections of the mean longitudes are made by means of epicycles, the magnitudes of which vary in different parts of the orbit. The dimensions are expressed in arcs of the orbits to which they belong, as follow (vv. 34-38):—

PLANET.	Circumference of Epicycle				
	Of Apsis			Of Conjunction	
	At even Quadrant.	At odd Quadrant.		At even Quadrant.	At odd Quadrant.
	°	°	′	°	°
Sun	14	13	40	..	..
Moon ..	32	31	40	..	..
Mercury ..	30	28	0	133	132
Venus ..	12	11	0	262	260
Mars ..	75	72	0	235	232
Jupiter ..	33	32	0	70	72
Saturn ..	49	48	0	39	40

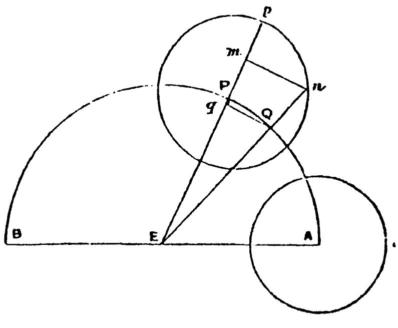
Let e_0, e_1, be the magnitudes of the epicycle at the even and odd quadrants respectively, expressed as above, $\delta\epsilon$ the correction to be applied to e_0 for any point whose mean anomaly is nt; let R = radius of circular orbit; then—

$$\frac{\delta\epsilon}{e_1 - e_0} = \frac{\text{sin.}\ nt}{\text{R}},$$

and the circumference of the epicycle e at that point is—

$$e = e_0 - \frac{\text{sin.}\ nt}{\text{R}}(e_1 - e_0).$$

The following is the calculation of the correction for the apsis (v. 39). Let E be the Earth, and A P B the undisturbed circular orbit; A a the radius of the epicycle at A, P p the radius of the epicycle at P; draw $n\ m$, Q q, $\perp^{r}$ to E p. The movement is represented by supposing the epicycle to move with its centre on the circle A P B, without revolving about its centre. Consequently, the radius A a, or its equivalent P n, is parallel to E A; in other words, $\angle\ p\ \mathrm{P}\ n = \angle\ \mathrm{P\ E\ A} = \theta$. The point Q, in which E n cuts the circle A P B, is the true position of the planet; and P Q is the correction sought.



Since the circumferences of circles are as their radii,—

$$\frac{360^\circ}{e} = \frac{\text{sin. Q}}{\text{sin. } p \text{ P } n};$$

And when, as in the case of the epicycles of the apsis, e is small, we have approximately—

$$m\ n = q\ \mathrm{Q} = \text{arc P Q}.$$

Hence the correction for the apsis—

$$\delta\ \theta = \frac{e}{360^\circ} \text{ sin. } \theta.$$

But if, as in the case of the conjunction, the epicycle is not small, we have (vv. 40-42)—

$$\frac{\mathrm{E}\ n}{n\ m} = \frac{\mathrm{E\ Q}}{\mathrm{Q}\ q}, \quad \mathrm{E}\ m^2 + m\ n^2 = \mathrm{E}\ n^2.$$

Hence, if $\delta_1\ \theta$ be the correction for the conjunction,—

$$\text{sin. } \delta_1\ \theta = \mathrm{Q}\ q = \frac{m\ m}{\sqrt{\mathrm{E}\ m^2 + m\ n^2}}\ \mathrm{R}.$$

The correction for the apsis is the only one required for the Sun and Moon. For the other planets, calculate (1) the correction of conjunction, and apply half of it to the mean place; thence (2) calculate that of the apsis, and apply half of it to the place already corrected; thence (3) calculate that of the apsis afresh, and apply it to the original mean place of the planet; and lastly, thence (4)

calculate that of the conjunction, and apply it to the last place (vv. 43-45).

The part of the equation of time, depending upon the difference between the Sun's mean and true places, is given in minutes by the formula (v. 46)—

$$\frac{\odot\text{'s equation} \times n}{3600'}$$

To calculate the correction $\delta\, n$ of the mean daily motion n of a planet due to the influence of the apsis; let ν be the mean motion of the apsis, then—

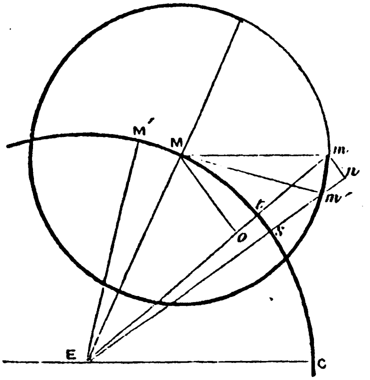
$$\delta \text{ sin. } \theta = \frac{(n - \nu)\ \delta \text{ tabular sin. } \theta}{225'};$$

Also, if ϑ be the arc of the epicycle (e), corresponding to θ in the orbit,—

$$\frac{360^\circ}{e} = \frac{\delta \text{ sin. } \theta}{\delta \text{ sin. } \vartheta} = \frac{\delta \text{ sin. } \theta}{\delta\, n};$$

Whence—

$$\delta\, n = \frac{e}{360} \frac{(n - \nu)\ \delta \text{ tab. sin } \theta}{225}.$$



To calculate the corresponding correction $\delta_1\, n$, due to the influence of conjunction (vv. 50, 51). In the following figure—

Let E be the Earth,
M′ M the correct daily motion of the centre of the epicycle at M′,
$m'\, m$ points in the epicycle corresponding to $m'\, m$.
Make $\text{E}\, n = \text{E}\, m$;
Join $\text{M}\, m$, and $\text{M}\, m'$;

Then in the $\triangle$s $\text{M}\, m\, t$, $\angle\, \text{M}\, t\, m = rt\ \angle$, $\therefore \text{M}\, m\, t = 90^\circ - m'\, m\, t$

$m\, c'\, m'$, $\angle\, m\, o'\, m' = rt\ \angle = m\, m'\, o$,

$\therefore$ the $\triangle$s are similar.

And since the epicycle is small compared with the orbit, we have approximately $t\, s = o'\, m'$.

$$\therefore \frac{o'\, m'}{m\, m'} = \frac{t\, m}{\text{M}\, m} = \frac{t\, s}{m\, m'},$$

$$\therefore \frac{t\, s}{t\, m} = \frac{m\, m'}{\text{M}\, m} = \frac{\text{M M}'}{\text{E M}}.$$

The text, however, substitutes E m for E M in the above expression without explanation; so that—

$$\delta_1 n = \text{M M}' \frac{t\ m}{\text{E}\ m}.$$

When the commutation in the final process of 43-45 is between the following limits, the motion becomes retrograde (vv. 52-55):—

Mercury	..	144°	to	215°
Venus ..	..	163°	,,	197°
Mars ..	..	164°	,,	426°
Jupiter ..	..	130°	,,	230°
Saturn ..	..	115°	,,	245°

To find the latitude of a planet (vv. 56-58). Subtract from the mean place of the planet, corrected for the apsis only, that of its node; then, if—

V = distance of planet from its node,
L = extreme, or greatest latitude,
l' = latitude at mean distance R,
l = ,, true ,, r;

We have for the Moon—

$$l = l^1 = \frac{\text{L sin. V}}{\text{R}};$$

And for the other planets—

$$\frac{r}{\text{R}} = \frac{l'}{l}, \text{ whence } l = \frac{\text{L sin. V}}{r}.$$

This is to be added to or subtracted from the declination, neglecting the difference between arcs measured on secondaries to the equator and ecliptic.

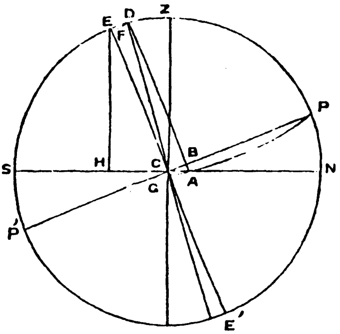
The day and night of a planet, or the interval of its passing twice over the same meridian, differs from a sidereal day and night by a quantity thus found. Each sign contains 1800′; then, if p = number of respirations occupied by the passage of the sign, in which the planet is, across the meridian (determined in chapter iii), the quantity required (v. 59)—

$$\frac{np}{1800'}.$$

The radius of the hour circle (v. 60)

$$= \text{R cos. D} = \text{R} - \text{R vers. D}.$$

To find the length of the day of a planet, or the time during which it is above the horizon (vv. 61-63). In the following figure—



Let N P Z S represent the meridian of an observer at C,

P P′ the N. and S. poles,

E E′ the points of the meridian cut by the equator.

E D the declination of the planet.

Draw D B A parallel to E C,

E H $\perp^r$ to N S,

P A G the arc of a great circle through P and A.

Then the Δ s, A B C, C H E, are similar.

$$\therefore \frac{EH}{HC} = \frac{CB}{BA} = \frac{g}{s}.$$

And— $\quad CB = DF = \text{sin. } D.$

Hence— $\quad AB = \dfrac{S \text{ sin. } D.}{g}.$

But the arc, of which A B is the sine, is the same part of the diurnal circle that the arc, of which C C is the sine, is of the equator.

$$\therefore \frac{AB}{BD} = \frac{GC}{CE'}$$

which determines G C. And the arc, of which G C is the sine, is the measure in time of the difference between a quadrant and the arc of a diurnal circle intercepted between the horizon and meridian.

The ecliptic is divided into 27 lunar mansions, each of which consequently contains 800′. Hence, in order to find in what mansion a planet is at any given time, let θ = its longitude.

No. of complete mansions traversed $= \left(\dfrac{\theta}{800'}\right)_q.$

Portion traversed by current mansion $= \left(\dfrac{\theta}{800'}\right)_r.$

No. of days elapsed $= \dfrac{1}{n}\left(\dfrac{\theta}{800'}\right)_r.$

A lunar day is $\frac{1}{30}$ of a lunar month, or of the interval in which the moon gains 360° in longitude on the Sun. It is therefore measured by $\frac{360}{30}° = 12° = 720'$. Hence we may find the number of lunar months and days elapsed by proceeding as above with a divisor 720′.

The yoga (योगः) (y) is the period during which the longitudes of the Sun (o) and of the Moon (o_1) together amount to the space of a lunar mansion (v. 65).

$$\therefore \frac{o_1 + o}{800'} = \text{No. of yogas passed} + \text{portion of current yoga}$$

$$= qy \qquad + z \text{ suppose.}$$

Then if n, n_1, be the daily motions—

$$\frac{60z}{n_1 + n} = \text{nádís elapsed of current yoga.}$$

Similarly for the lunar days (d_1)—

$$\frac{o_1 - o}{720} = qd_1 + z,$$

$$\frac{60z}{n_1 - n} = \text{nádís elapsed of current lunar day.}$$

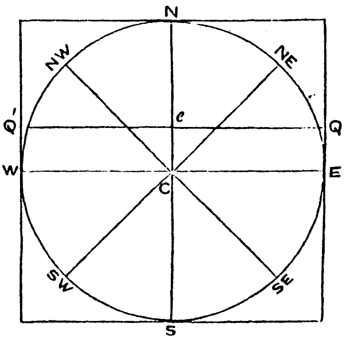
Each lunar day is divided into two halves (करणाः), which have particular names and portions assigned to them. But they appear to have no practical use.

Chapter III.

On Direction, Place, and Time.

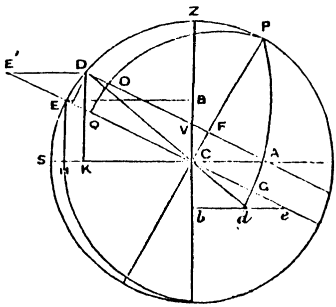
To construct the dial (vv. 1–7).

Describe a circle; at its centre erect a gnomon 12 digits high. Mark the two points where in the forenoon and afternoon the extremity of the shadow meets the circle. From each point as a centre, with the distance between them as a radius, describe two circles; the line joining their points of section will be a N and S line. By similar processes draw E and W, NE and SW, NW and SE lines. Draw a circumscribing square, and mark off the sides passing through the E and W points in digits. Let e be the extremity of the shadow at noon, when the Sun is on the equinox; through e draw Q Q′ ∥ to E W.



Then (v. 8) if g = height of gnomon,
h = length of hypothenuse,
s = „ equinoctial shadow,

$$h^2 = g^2 + s^2.$$



In a yuga the sidereal circle lags behind on the zodiac 600 revolutions. And the position x of the initial point of the sphere at any period is consequently given by the following proportion:—

When d = No. of days elapsed,
d_1 = „ in a yuga,

$$\frac{d_1}{d} = \frac{600 \text{ rev.}}{x}$$

But it is an oscillatory movement and has periods and changes of sign like a sine; only the range instead of being 90° is

$$\frac{90° \times 30}{10} = 27°.$$

The annual precession consequently

$$= \frac{365 \times 360° \times 360}{4320000 \times 365} \frac{3}{10} = 54''.$$

In the accompanying figure let (vv. 12–25)

C b represent the gnomon $= g$,
$b\ e$ „ equinoctial shadow $= s$,
$b\ d$ „ any other „ $= s'$,
C e „ equinoctial hypothenuse $= h$,
C d „ any other „ $= h'$,
C E „ radius = R,
$d\ e$ „ measure of amplitude $= a_1$,
Z „ zenith,
S „ south point.

Then if the Sun be upon the equator—

$$\sin. l = \text{B E} = \frac{\text{R}\, s}{h},$$

$$\cos. l = \text{C B} = \frac{\text{R}\, g}{h}.$$

If it be not on the equator—

$$\sin. \text{Z} = \text{B E},$$
$$\sin. \text{A} = \text{C B},$$
$$\text{and } 90° - \text{L} = \text{Z} \pm \text{D}.$$

Similarly from the above equations we may find the shadow or hypothenuse for a given latitude or declination.

The true and mean longitudes may be found, when the latitude and declination are known, by an inversion of the processes of ii 28–30.

If a represent the Sun's amplitude, then C A = sin. a; and from the similar $\triangle$s C b c, C F A, we have—

$$\text{sin. } a = \frac{h \text{ sin. D}}{g}.$$

Also from the similar $\triangle$s C d e, D C A, we have—

$$a_1 = \frac{h \text{ sin. D}}{g} \frac{h'}{\text{R}};$$

$$\text{and } s' = s \pm a_1.$$

Returning to the figure—

Z C will be the projection of the prime vertical,

V " " point where the Sun passes it.

Then V C = sin. A.

And generally $\text{sin. A} = \frac{\text{R } g}{h}.$

Now since the $\triangle$s V C F, E C H are similar,

$$\text{sin. A} = \frac{\text{R sin. D}}{\text{sin. } e}.$$

Hence if h'' be the value of the hypothenuse when the sun is on the prime vertical—

$$\frac{\text{sin. D}}{\text{sin. } e} = \frac{g}{h''}.$$

Also since—

$$\frac{\text{sin. } l}{\text{cos. } l} = \frac{s}{g}$$

$$\therefore h'' = \frac{s \text{ cos. } l}{\text{sin. D}}.$$

Again, since the $\triangle$s H E C, F C A are similar—

$$\text{sin. } a = \frac{\text{R sin. D}}{\text{cos. } l}.$$

Whence (v. 28) $a_1 = \frac{h^1 \text{ sin. } a}{\text{R}}.$

To find, for a given declination and latitude, the sine of the Sun's altitude at the moment when it crosses the SE or the SW vertical circle.

Suppose that the plane of the above figure is turned about C Z as an axis through 45°, so that Z D E S represents an arc of SE, or

of a SW circle. Then $b\,e = \sqrt{2}\,s$, $D\,E' = \sqrt{2}\,\sin.\,a$. The altitude is then given by the formula—

$$\sin.\,A = \pm\,\frac{2\,s\,g\,\sin.\,a}{g^2 + 2\,s^2} + \sqrt{\frac{(R^2 - 2\,\sin.^2 a)\,g^2}{g^2 + 2\,s^2} + \frac{4\,s^2\,g^2\,\sin.^2 a}{(g^2 + 2\,s^2)^2}}.$$

To verify this, form the quadratic equation in sin. A—

$$(g^2 + 2\,s^2)\,\sin.^2 A - 4\,s\,g\,\sin.\,a\,\sin.\,A + 2\,g^2\,\sin.^2 a - R^2 g^2 = 0,$$

or

$$2\,(s\,\sin.\,A - g\,\sin.\,a)^2 - g^2\,(R^2 - \sin.^2 A) = 0,$$

or, referring to the figure—

$$(b\,e\,.\,D\,K - C\,b\,.\,D\,E')^2 - C\,b^2\,C\,K^2 = 0.$$

But

$$\frac{C\,K + D\,E'}{D\,K} = \frac{b\,e}{C\,b},\ \text{or}\ b\,e\,.\,D\,K - C\,b\,.\,D\,E' = C\,K\,.\,C\,b,$$

which renders the equation identical.

If the declination be south, D E′ falls in the opposite direction: hence the double sign.

To find the sine of the Sun's altitude at any hour, when its distance from the meridian, the declination, and the altitude are known (vv. 34-36).

In the same figure as before, let O be the projection of the Sun's place at any time, and P O Q that of an arc of a great circle; P A G that of another.

Then—

C G = sine of ascensional difference = sin. a.
E Q = versine of hour angle = versin II.
O R = sin. A.
R_1 = radius of diurnal circle.

Then—

$$\sin.\,A = (R + \sin.\,a - \text{versin}\,H)\,\frac{R_1}{R}\,\frac{\cos.\,l}{R}$$
$$= \frac{G\,Q\,.\,F\,D\,.\,E\,H}{C\,E^2}.$$

But—

$$\frac{E\,H}{C\,E} = \frac{O\,R}{O\,A}\ \text{and}\ \frac{D\,F}{C\,E} = \frac{G\,Q}{A\,O}\quad \therefore\ \frac{G\,Q\,.\,F\,D\,.\,E\,H}{C\,E^2} = O\,R.$$

In a given latitude to find the Sun's declination, and thence its

true and mean longitude from the shadow at any hour (vv. 40-61). It was shown above that—

$$\frac{h}{a} = \frac{\text{E C}}{\text{C A}} = \frac{\text{E H}}{\text{F C}}$$

$$\therefore \text{F C} = \sin.\ \text{D} = \frac{a \cos.\ l}{h}.$$

To describe on the dial the path of the extremity of the shadow for any day, set off three bases in the forenoon, noon, and afternoon (y-co-ordinates); calculate the distances E. and W. (x-co-ordinates); and draw a circle through the three points. This represents the path required (vv. 41, 42).

To determine the time occupied by each sign in rising. First for a point on the equator. Let R_1, R_2, R_3 be the day radii of 1, 2, 3 signs respectively. Then if t_1, t_2, t_3, be their time of rising, and s_1'', s_2'', s_3'' the number of seconds in 30°, 60°, 90° respectively; then—

$$\sin.\ t_1 = \frac{\text{R}_3 \sin.\ s_1''}{\text{R}_1},\quad \sin.\ t_2 = \frac{\text{R}_3 \sin.\ s_2''}{\text{R}_2},\quad \sin.\ t_3 = \frac{\text{R}_3 \sin.\ s_3''}{\text{R}_3}$$

For the two sets of quantities (s_1'', s_2'', s_3''), (t_1, t_2, t_3), being measured, one on the ecliptic, and the other on the equator, form respectively the hypothenuses and bases of three rt. $\angle$d $\triangle$s; and by ordinary spherical trigonometry—

$$\sin.\ s'' = \cos.\ s_1''t_1 \sin.\ t_1 = \frac{\text{R}_1}{\text{R}_3} \sin.\ t_1.$$

For the next three signs the expressions will be the same as these, only in an inverse order; and so on for the other six. For a point not on the equator we must add (or subtract) the ascensional difference.

Given the Sun's longitude and the local time, to find the points of the ecliptic on the horizon and on the meridian (vv. 46–49).

The preceding method gives the ascensional equivalents for the various signs; and for the portions of signs, in which either the Sun or the horizon is, we have $\dfrac{\delta\ s_2''}{\delta\ t_2} = \dfrac{s''}{t_2}$.

A similar process will give the local time at which any given point on the ecliptic will be upon the horizon (vv. 50, 51).

CHAPTER IV.

ON ECLIPSES.

The diameter of the Sun $= 6{,}500$ yojanas (v. 1).

" Moon $= 480$ "

Let, as before, n, n_1, represent the mean, and n' n_1', the true daily motions of the Sun and Moon respectively; then—

$$\text{Corrected diameter of the Sun} = 6500\,\frac{n'}{n},$$

$$\text{"} \quad \text{Moon} = 430\,\frac{n_1'}{n_1}.$$

In order to find the apparent diameters in minutes of arc, the corrected diameter of the Sun is projected on a circle at the Moon's mean distance, by multiplying the expression by the ratio of the Sun's revolutions in an age to those of the Moon, or by that of the Moon's to the Sun's orbit. At the distance in question $1' = 15$ yojanas (vv. 2–3).

$$\text{Corrected diameter of the Earth} = 1600\,\frac{n_1'}{n_1}.$$

To find the diameter of the Earth's shadow upon the Moon's mean orbit. Project the difference of the Sun's and the Earth's corrected diameters on the Moon's orbit, and subtract the result from the Earth's corrected diameter (vv. 4, 5). The formula is—

$$1600\,\frac{n_1'}{n_1} - \left(6500\,\frac{n'}{n} - 1600\,\frac{n_1'}{n_1}\right)\frac{480}{6500}.$$

Calculate the longitudes of the Sun and Moon at midnight next preceding or following the opposition or conjunction; then, if an eclipse be probable, calculate the interval to the instant of opposition or conjunction, by the methods of Chapter II (vv. 6–9).

If r_1, r, be the radii of the eclipsed and eclipsing bodies, and l_1 the latitude of the former, the amount of the obscuration is given by the formula (vv. 6–11)—

$$r_1 + r - l_1.$$

The times of duration of the eclipse, and of total obscuration will be expressed by (vv. 12, 13)—

$$2\sqrt{(r_1 \pm r)^2 - l^2} \times \frac{60}{n_1 - n'},$$

60 being the number of nádís in a day.

This method assumes that the latitude remains unchanged during the eclipse; but if greater accuracy is required, with the above formula as a first approximation recalculate both longitude and latitude of the Moon; and repeat the process as often as desired (vv. 14, 15).

The instant of true opposition or conjunction is considered as the middle of the eclipse (vv. 16, 17).

If from the formula of vv. 12, 13, corrected by vv. 14, 15, we subtract any interval of time (t), and reconvert the remainder into arc, we may regard the result as the perpendicular, and the latitude as the base of a right-angled triangle; the hypothenuse will then represent the amount of obscuration at the time t. In the case of a solar eclipse, a correction for parallax during its continuance must be made. This is explained by Chapter V, vv. 14–17 (vv. 18–21). Conversely we may require to know when the obscuration will attain to a given amount. The method, being similar to those given above, need not be given in detail (vv. 22, 23).

In projecting an eclipse (a process which is explained in Chapter VI), the eclipsed body is represented in the centre of the figure with a N. and S. line, and an E. and W. line drawn through it as co-ordinates, or lines of reference. The N. and S. line represents a great circle drawn through the N. and S. points of the horizon; the E. and W. line a small circle parallel to the prime vertical. The position of the ecliptic is fixed by calculating, first, the angle (v) between the E. and W. line and the circle of diurnal motion; and secondly, the angle (w) between the latter circle and the ecliptic.

For the first process let P be the pole of the equator, M the eclipsed body, N the north point of the horizon. Then

$$\begin{aligned} \mathrm{P\,N} = l,\ \ \mathrm{M\,P\,N} &= 180^\circ - \text{hour angle} \\ &= 180^\circ - \mathrm{H}. \end{aligned}$$

If M were on the horizon and at the E. or W. point, then P M and N M would be quadrants, and P M N $= v$. It is, however, assumed that the same relations would remain approximately unchanged for other positions of M; hence for the triangle P M N, we have—

$$\frac{\sin.\ \mathrm{M\,P\,N}}{\sin.\ \mathrm{M\,N}} = \frac{\sin.\ \mathrm{P\,M\,N}}{\sin.\ \mathrm{P\,N}},\ \text{ or }\ \sin.\,v = \frac{\sin.\ \mathrm{H}\ \sin.\ l}{\mathrm{R}}.$$

Secondly, it seems supposed that the diurnal circle and the equator meet at 90° from M; hence moving M to a point M′, 90° forward

on the ecliptic, the declination of M′ will measure the angle between the ecliptic and the diurnal circle.

The sine of the deflection $(v+w)$ so found is laid off on a straight line on the scale of radius = 49 digits; *i.e.*, 3438 ÷ 49 = 70, or 70′ = 1 digit.

To take account of the apparent increase of heavenly bodies near the horizon; it is assumed that 3′ at the horizon are equivalent to 4′ at the zenith. Hence it is calculated (v. 26)—

$$\frac{\frac{1}{2}\text{ day}}{1'} = \frac{\text{altde. in time}}{\text{excess over } 3'}$$

Whence the rule—

$$\text{Equivalent of digits in minutes of arc} = \frac{\text{alt. in time} + 3\tfrac{1}{2}\text{ days}}{\frac{1}{2}\text{ day}}.$$

Chapter V.

On Parallax in a Solar Eclipse.

When the Sun is on the meridian, it is considered that there is no parallax in longitude. When the latitude is equal to the declination, there is none in latitude (v. 1).

The first step towards finding the parallax at the moment of conjunction, is to determine the sine of amplitude of the point of the ecliptic on the eastern horizon (vv. 2, 3). For this purpose, let D_1 = the greatest declination; then, adopting the notation hitherto used, we have by ii, 28—

$$\text{R sin. D} = \text{sin. } \theta \text{ in. s}D_1,$$

And by iii, 22, 23—

$$\text{sin. } a = \frac{\text{R sin. D}}{\text{cos. } l};$$

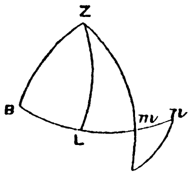
Whence—

$$\text{sin. } a = \frac{\text{sin. } \theta \text{ sin. } D_1}{\text{cos. } l}.$$

To find the sines of the Z D and altitude of the point of the ecliptic having the greatest altitude (vv. 4-6),—

Let $Z' =$ meridian Z D,
$C_1 =$ Z D of point in question,
$A_1 =$ Altitude ,,

Then if, in the spherical triangle Z B L, Z be the zenith, Z L a meridian, Z L $\perp$ to the ecliptic B L, and M n the arc of a great circle from M to the pole of the ecliptic,



$Z\,L = Z'$,
$Z\,B = Z_1$,
$Z\,L\,B = 90^\circ - B\,Z\,L$, approximately,
$m\,n =$ parallax in longitude,

And—

$$\frac{\sin.\ z_1}{\sin.\ z'} = \frac{\sin.\ Z\,L\,B}{\sin.\ Z\,B\,L} = \frac{\sin.\ Z\,L\,B}{R} = \frac{\sin.\ (90^\circ - a)}{R} = \frac{\sqrt{R^2 - \sin.^2 a}}{R},$$

And— $$\sin.^2 Z_1 + \sin.^2 A_1 = R;$$

Whence—

$$\sin.\ Z_1 = \sqrt{\sin.^2 z' - \frac{\sin.^2 z'\ \sin.^2 a}{R^2}}$$

$$\sin.\ A_1 = \sqrt{R^2 - \sin.^2 Z_1}.$$

But (v. 7) we may approximately take $Z_1 = z'$.

To find the parallax in longitude (vv. 7-9). The Moon's greatest horizontal parallax = 4 nádís. Hence the proposition—

$$\frac{\sin.\ Z\,m}{M\,m} = \frac{R}{4}.$$

But— $$\frac{\sin.\ Z\,m}{M\,m} : \frac{\sin.\ B\,m}{n\,m} = \frac{\sin.\ A_1}{R}$$

$$\therefore \frac{\sin.\ B\,m}{n\,m} \cdot \frac{\sin.\ A_1}{R} = \frac{R}{4}.$$

But $\frac{R}{2} = \sin.\ 30^\circ$; hence—

$$\text{Parallax in longitude} = \frac{\sin.\ B\,m}{\sin.\ 30^\circ \div \sin.\ A_1}.$$

[The term used in the text seems rather to imply L m, instead of B m.]

The formula must be used to correct the time of conjunction previously found; the parallax must then be calculated afresh, and the process repeated.

To determine the parallax in latitude (v. 10), we have only to substitute, from the formulæ—

$$\text{Sun's greatest parallax} = \frac{n}{15}$$

$$\text{Moon's} \quad \text{,,} \quad = \frac{n_1}{15},$$

the quantity $\frac{n_1 - n}{15}$ for 4 in the equation $\frac{\sin. Z\, m}{M\, m} = \frac{R}{4}$; whence

$$\text{Parallax in latitude} = \frac{(n_1 - n)\ \sin. Z_1}{15\ R}.$$

This formula may be simplified for calculation by the following considerations (v. 11):—

$$n_1 - n = 731'\ 27''$$
$$15\ R = 51570',$$

and—

$$\frac{51570'}{731'\ 27''} = 70\tfrac{1}{2} = \frac{R}{49}\ \text{nearly.}$$

Hence, approximately—

$$\text{Parallax in latitude} = \frac{\text{Sin.} Z_1}{70},\ \text{or} = \frac{49\ \text{Sin.} Z_1}{R}.$$

With the value of the parallax so found, the time of conjunction is to be corrected. The parallaxes in longitude (p_1, p_2, p_3) for the beginning, middle, and end of the eclipse respectively having been calculated; the quantities—

$$p_2 \pm p_1, \quad p_2 \pm p_3$$

are to be added as corrections to the half durations previously determined.

Chapter VI.

On Planetary Conjunctions.

To find when two planets will have the same longitude (vv. 3-6).

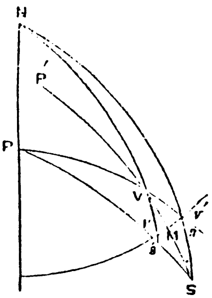
Let θ', θ'_1, be their longitudes,

n, n_1, their daily motions.

Then they are distant from the point where they will have the same longitude, respectively,—

$$\frac{(l - l_1)\,n}{n \pm n_1}, \text{ and } \frac{(l - l_1)\,n}{n \pm n_1}.$$

To find the moment of conjunction, *i.e.*, when they will be on the same secondary to the ecliptic (vv. 7-12).



Let V, S, be the two planets having the same longitude,

N the north point of the horizon,

P, P', the poles of the equator and ecliptic,

P S, P V, great circles from P, cutting the ecliptic in *s* and *v*,

N S, N V, great circles from P, cutting the ecliptic in *s'* and *v'*,

then the two planets are removed from conjunction by the distance *v s*. To determine this, find—

$$\text{M}\,v + \text{M}\,s = v\,v' - \text{M}\,v' + s\,s' - \text{M}\,s'.$$

Let ss'_0 be the value of $s\,s'$ when S is on the horizon. Then P S M is an angle which $=$ 0° when the pole is on the horizon, and $=$ 90° when it is at the zenith; and is on that account supposed to vary with the elevation of the pole; in other words, it is assumed $=$ the latitude (L) of the observer. And S being supposed always near the ecliptic, *s* S *s'* is regarded as a plane triangle, having the angle *s' s* S $=$ 90°. Hence—

$$\frac{s\,s'_0}{l} = \frac{\text{sin. L}}{\text{cos. L}} = \frac{s}{g}$$

Again, for any other position of S, we have the proposition—

$$s\,s' : s\,s'_0 = \text{merid. dist. in time} : \tfrac{1}{2}\text{ day},$$

or—

$$s\,s' = \frac{l\,s}{g}\,\frac{t}{\frac{1}{2}\text{ day}}.$$

In the same way, in the $\triangle$ M S *s'*, the angle at *s'* is considered as $=$ 90°, and consequently—

$$\frac{\text{M S}}{\text{M } s'} = \frac{\text{R}}{\text{sin. M S } s'}.$$

But since M is supposed always near the ecliptic, M S s' = P′ S P = P′ M P, nearly; and if D′ be the declination of a body 90° in advance of M,—

$$\frac{\mathrm{M\,S}}{\mathrm{M}\,s'} = \frac{\mathrm{R}}{\sin.\ \mathrm{D}'}.$$

But—

Sine of greatest declination = sin. 24° = 1397′ = 58 × 24′ nearly;
whence it is concluded that sin. D′ = 58 × D′:
also radius = 3438′ = 58 × 60′ nearly;

whence—

$$\mathrm{M}\,s' = \frac{l\ \mathrm{D}'}{60}.$$

In the same way $v\ v'$, and M v' may be found; and thence $v\ s$ completely determined.